The electromagnetic actuator design problem: An adapted interval global optimization algorithm using model reformulation and constraint propagation

I. Mazhoud*, K. Hadj-Hamou*, J. Bigeon* and G. Remy**
*G-SCOP - CNRS, Grenoble-INP-UJF, 46 Av. Félix Viallet, 38000 Grenoble, France
**Laboratoire de Génie Electrique de Paris (LGEP) / SPEE-Labs, CNRS UMR 8507; SUPELEC; Université Pierre et Marie Curie P6; Université Paris-Sud 11;
11 rue Joliot Curie, Plateau de Moulon F91192 Gif sur Yvette CEDEX, France
issam.mazhoud@grenoble-inp.fr, khaled.hadj-hamou@grenoble-inp.fr,
jean.bigeon@grenoble-inp.fr, ghislain.remy@lgep.supelec.fr

***Abstract* - This paper presents an adapted deterministic global optimization algorithm applied to the optimal design of electromagnetic actuators. The algorithm is based on interval arithmetic and constraint propagation. A new reformulation step is introduced in order to accelerate the convergence of the algorithm and increase the solutions accuracy. The tests have been performed according to three performance criteria: convergence, precision and number of iterations.**

I. INTRODUCTION

In the preliminary design of electromagnetic machines, a design model is usually dealt with. There are principally two sources to these models: models from response surface methods [1] (RSM) based on finite elements models and models from the physico-mathematical modeling of the device. A design model is the aggregation of a physico-mathematical model or RSM model and specifications (see Figure 1). The aim of the preliminary design phase is to propose a first quantification of the design parameters that will be the basis of the future prototypes. Consequently, proposing the best solution reduces prototyping costs. F. Messine has proved in [2] the interest of using global optimization methods in the design of electromagnetic actuators, the use of such methods provides a gain of about 10%. In addition, in some cases, the global optimum is strongly required. Table 1 summarizes the principal global optimization approaches available considering the structure of the constraints (non linear and non convex); the stochastic approach (Particle Swarm Optimization (PSO), simulated annealing…), sometimes coupled with local optimization techniques (Quasi-Newton…) and the Interval Branch and Bound Algorithm (IBBA) which is a deterministic global optimization approach.

	Stochastic	Stochastic + Local Deterministic	IBBA
Derivatives computation	No	Yes	Yes
Rewrite the algebra	No	No	Yes
Accepted models	Black box	Black Box (if derivatives provided) and White box (else)	White box
Optimum proof	No	No	Yes

Table 1.Possible optimization approaches

4 criteria are chosen to guide the choice of our methodology; the need of the derivatives during the algorithm, whether or not it uses other arithmetic than the conventional arithmetic, the types of models accepted (black and/or white boxes) and whether or not it offers optimality proof.

As summarized in table 1, the stochastic methods have the advantage of simplicity in writing, unlike the IBBA, which requires a rewriting of the model or at least a recompilation as it uses the interval arithmetic [3]. However, the rewriting step can be automated like in some software [4]. In addition, the stochastic approaches do not offer proofs of optimality if a solution is found or proofs of non-existence if not. Moreover, the interval arithmetic is a very suitable approach to address problems in electromagnetic design, as the design parameters generally evolve in continuous intervals [5]. Given that in some cases, a proof of optimality or of non-existence of solutions is required, the IBBA has been chosen.

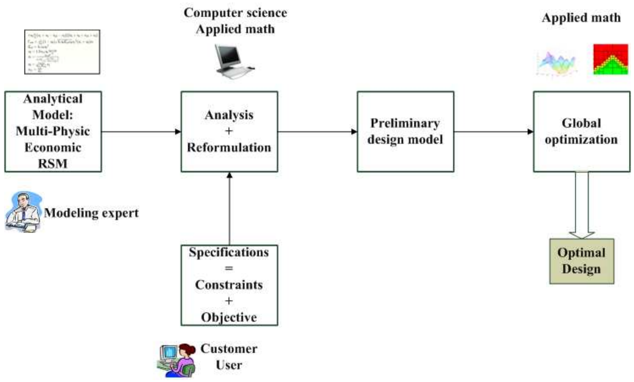

Figure 1.Preliminary design approach: reformulation and optimization

In this paper, we introduce an adapted global optimization algorithm that aims at providing exact solutions for the optimal design of electromagnetic actuators if the preliminary design model is feasible, and proof of non existence if not. This method is based on extended interval analysis and constraint propagation [3] coupled with a new reformulation.

II. OPTIMIZATION ALGORITHM AND REFORMULATION

The algorithm uses the branch and bound structure: the branching is performed by bisecting the search domain (hypercube) and the bounding by evaluating the design model and discarding the boxes that do not contain the global optimum. During the algorithm execution, an upper bound of the global minimum is updated every time a better feasible solution is found. This upper bound is initialized to positive infinity. The performance criteria that will be used to evaluate the algorithm are the number of model evaluations necessary to find the global optimum, and the iteration duration. For more details about the algorithm, please refer to [3], [6] and [7].

In the previous algorithms [6], the equality constraints were generally transformed into inequalities in order to make the updating step easier. For example, if we decide to relax the constraint $h(X) = 0$ by ε (positive), the new constraint would be $-\varepsilon \leq h(X) \leq \varepsilon$. Knowing that the constraints of the physico-mathematical model are the expression of physical laws, it is not relevant to relax them.

When we first implemented the algorithm, we noticed that the constraints had to be relaxed in order to make the algorithm converge easier, and even so, the number of iterations remained high. In addition, we noted that finding a feasible combination to update the upper bound was very difficult. This is because the choice of the potential feasible point is done independently of the constraints. In order to handle this problem, a specific reformulation step is introduced. This reformulation aims firstly at accelerating algorithm convergence by reducing the number of model evaluations, and secondly at improving the solution's relevance as it permits to obtain exactly feasible solutions. It consists in reorganizing the equations, reformulating them, and defining the links between the parameters in order to classify them as inputs and outputs, though we deal with the inputs only (the outputs are the consequences of the input choices).

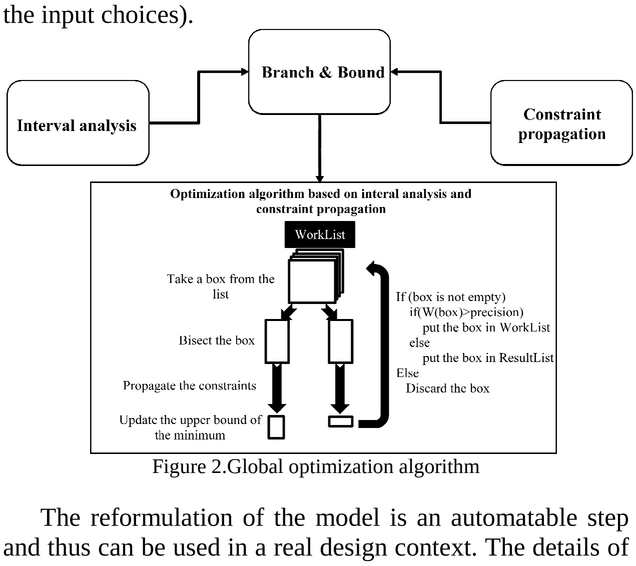

Figure 2.Global optimization algorithm

The reformulation of the model is an automatable step and thus can be used in a real design context. The details of the optimization algorithm and the reformulation procedure will be provided in the full paper article.

III. PRINCIPAL RESULTS

The algorithm has been tested on the electromagnetic actuators design model. The model and specifications are developed in [2]. In order to numerically measure the efficiency of the reformulation step introduced here, we present in table 2 the evolution of the solution during the execution of the algorithm and the corresponding numbers of model evaluations and durations.

$$\begin{cases} \min\ V_u = \pi \frac{D}{\lambda}(D + E - e - l_a)(2C + E + e + l_a) \\ D = \frac{p\Delta_p}{\pi} \\ J_{cu} = \sqrt{\frac{E_{ch}}{k_r E}} \\ B_e = \frac{2 l_a M}{D \log\left[\frac{D + 2E}{D - 2(l_a + e)}\right]} \\ K_f = 1.5 p \beta \frac{e + E}{D} \\ \lambda = \frac{\pi}{2\Gamma_{em}}(1 - K_f)\sqrt{k_r \beta E_{ch} E}\, D^2 (D + E) B_e \\ C = \frac{\pi \beta B_e}{4 p B_{fer}} D \end{cases} \quad \begin{cases} k_r = 0.7 \\ B_{fer}(T) = 1.5 \\ E_{ch}(A/m) = 10^{11} \\ \Gamma_{em}(N.m) = 10 \\ M(T) = 0.9 \\ \Delta_p(m) = 0.1 \end{cases} \quad \begin{cases} Bound \\ constraints \end{cases}$$

Figure 3.Electric actuator design optimization model

Without reformulation				With reformulation		
Solution (1 e-4)	Duration (s)	Iterations		Solution (1 e-4)	Duration (s)	Iterations
Not solved				10.76	0.04	4
				8.55	0.076	17
				6.67	0.368	1486
				6.15	0.521	2705
				6.073	16.936	133282

Table 2.Evolution of the solution during the algorithm execution

In addition to the reduced number of model evaluations, the solutions found are very accurate as they scrupulously respect the constraints. Table 2 shows that the global minimum is reached after 133282 iterations, which corresponds to 16.9 seconds. We notice that without the reformulation, the algorithm was unable to converge, as the upper bound has never been updated (duration>1 hour). We tested the same algorithm, without reformulation but with relaxed constraints ($\varepsilon = 10^{-4}$, which corresponds to 2.5%), and we were able to find the same optimum after almost 19 millions iterations (45 minutes).

Comparative tests with a stochastic algorithm based on constrained PSO, on other engineering design problems with both formulated and non-formulated models, will be introduced and discussed in the full paper article.

IV. REFERENCES

[1] P. Alotto, M. Gaggero, G. Molinari, and M. Nervi, "A 'design of experiment' and statictical approach to enhance the 'generalized response surface' method in the optimization of multiminima problems," *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 1896-1899, 1997.

[2] F. Messine, B. Nogarede, and J. L. Lagouanelle, "Optimal design of electromechanical actuators: A new method based on global optimization," *IEEE Trans. Magn.*, vol.34, pp. 299-308, 1998.

[3] E. Hansen, and G. W. WALSTER, "Global Optimization Using Interval Analysis," *Marcel Dekker*, 2004.

[4] http://www.designprocessing.com/

[5] G. Saxena, and D. A. Lowther, "The use of interval mathematic in electromagnetic design," *IEEE Trans. Magn.*,vol. 37, no. 5, pp3588-3591, 2001.

[6] F. Messine, "Deterministic global optimization using Interval Constraint Propagation Techniques," *RAIRO Oper. Res*, vol.38, pp. 277–293, 2004.

[7] E. Fitan, F. Messine and B. Nogarede, "The Electromagnetic Actuator Design Problem: A General and Rational Approach," *IEEE Trans. Magn.*, vol.40, no. 3, pp. 1579-1590, 2004.